

## Encontro [CAMGSD](#) - [CMAFCIO](#) de Equações Diferenciais

14 e 15 de setembro, 2017

FCUL, sala 6.2.33

### Programa

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O CAMGSD e o CMAFCIO organizam um Encontro de Equações Diferenciais que se realizará na Faculdade de Ciências da Universidade de Lisboa na semana de acolhimento de setembro de 2017, nas tardes de 14 e 15. Este encontro conjunto tem como âmbito a área alargada das equações diferenciais e destina-se a promover o contacto entre os investigadores das duas unidades que trabalham nesta área científica, sendo aberto a todos os interessados. O programa previsto compreende intervenções de investigadores de ambos os centros de acordo com o seguinte horário:

	Dia 14	Dia 15
14h00	José Francisco Rodrigues	Luís Barreira
14h45	João Alves	Fernando Pestana da Costa
15h30	Nicolas Van Goethem	Teresa Faria
16h15	(Café)	(Café)
16h30	Simão Correia	Pedro Girão
17h15	Henrique Oliveira	Hugo Tavares

**José Francisco Rodrigues** (CMAFCIO \ Faculdade de Ciências)

*On the obstacle-mass constraint problem for hyperbolic conservation laws*

In this work we introduce the obstacle-mass constraint problem for a multidimensional scalar hyperbolic conservation law. We prove existence of an entropy solution to this problem by a penalization/viscosity method. The mass constraint introduces a nonlocal Lagrange multiplier in the penalized equation, giving rise to a nonlocal parabolic problem. We introduce a compatibility condition relating the initial datum and the obstacle function which ensures global in time existence of solution. This is not a smoothness condition, but relates to the propagation of the support of the initial datum. Joint work with Paulo Amorim and Wladimir Neves.

**João Alves** (CAMGSD \ Instituto Superior Técnico)

*Rational criteria for diagonalizability*

As it is known, to decide whether a given  $n \times n$  matrix is diagonalizable is a basic problem in Linear Algebra and relevant in the most diverse areas of Mathematics. In the literature, the concept of minimal polynomial of a matrix is presented as the central element of the most well-known rational criterium for diagonalizability: a  $n \times n$  matrix, with coefficients in a field  $K$ , is diagonalizable if and only if its minimal polynomial,  $\mu(z) = z^m + c_{m-1}z^{m-1} + \dots + c_0$ , has  $m$  distinct roots in  $K$ . There are, however, other noteworthy rational criteria that recent literature, as far as we know, does not contemplate. These criteria and their relation to some important theorems of the theory of algebraic equations are the main ingredients of this discussion.

**Nicholas Van Goethem** (CMAFCIO \ Faculdade de Ciências)

*A variational approach to single crystals with dislocations*

A three-dimensional single crystal with dislocations forming general one-dimensional clusters is studied in the framework of finite elasticity. By virtue of a variational approach, a free energy depending on first- and second-order deformation fields is considered. Classical linear invariants and, to account for the dislocation micro-structure, a quadratic higher-order invariant are considered as model variables, as well as the dislocation density, identified with the deformation curl. The classical direct method of J. Ball for nonlinear elasticity is used in conjunction with the theory of integral currents first introduced by H. Federer. The problem we address is a joined minimization in the deformation field and the dislocation lines, bound by a geometric constraint.

**Simão Correia** (CMAFCIO \ Faculdade de Ciências)

*Local and global well-posedness for the nonlinear Schrödinger equation in spaces of infinite mass*

We consider the Cauchy problem for the classical nonlinear Schrödinger equation. The standard setting is to consider the initial data in some square-integrable space, since these are the

known spaces for which the linear group is actually an isometry. The goal of this talk will be to explore local and global well-posedness in spaces without square-integrability. The ideas and techniques involved are accessible to non-experts in the area and also general enough to be applicable to other dispersive equations.

**Henrique Oliveira** (CAMGSD \ Instituto Superior Técnico)

*Bifurcation equations for periodic orbits of implicit discrete dynamical systems*

Bifurcation equations, non-degeneracy and transversality conditions are obtained for the fold, transcritical, pitchfork and flip bifurcations for periodic points of one dimensional implicitly defined discrete dynamical systems obtained by discretization of PDE's and ODE's. The backward Euler method and the trapezoid method for numeric solutions of ordinary differential equations fall in the category of implicit dynamical systems. Examples of bifurcations are given for some implicit dynamical systems including bifurcations for the backward Euler method when the step size is changed.

**Luís Barreira** (CAMGSD \ Instituto Superior Técnico)

*Robustness of hyperbolicity*

I will discuss how one can establish the robustness of hyperbolicity, that is, its persistence under sufficiently small perturbations. This may include, for example, noninvertible evolution families and various flavors of hyperbolicity. The emphasis will be on presenting the main ideas, keeping technicalities to a minimum. This is joint work with Claudia Valls.

**Fernando Pestana da Costa** (CAMGSD \ Instituto Superior Técnico)

*Sub-monolayer deposition models: similarity profiles and convergence rates*

We present an infinite system of ODEs that is used in modelling the deposition of adatoms in a monolayer in a crystal facet, and we will study the convergence of its solutions to similarity profiles, including their convergence rates. The techniques used consist in the decoupling of the infinite system into a two-dimensional ODE and a linear triangular infinite ODE system; in the study of convergence rates of the two-dimensional system using center manifold analysis tools, and in the analysis of the behavior of the infinite triangular system by a fine analysis of the terms obtained by the application of the variation of constants formula.

**Teresa Faria** (CMAFCIO \ Faculdade de Ciências)

*Positive periodic solutions for a family of impulsive delay differential equations*

For periodic scalar delay differential equations (DDEs) of the form  $x'(t) + a(t)x(t) = g(t, x_t)$  subject to linear periodic impulses, the existence of a positive periodic solution is established under very general conditions on the nonlinearity  $g$  and on the impulses. Such periodic solutions are obtained as fixed points for an appropriate operator. The results are illustrated with applications to some important examples from mathematical biology, such as Mackey-Glass and Nicholson's blowflies equations, improving and generalizing other criteria in recent literature. For some classes

of non-autonomous  $n$ -dimensional DDEs, a criterion for the existence of a positive periodic solution will also be presented.

**Pedro Girão** (CAMGSD \ Instituto Superior Técnico)

*On the occurrence of mass inflation for the Einstein-Maxwell-Scalar field system with a cosmological constant and an exponential Price Law*

We study the spherically symmetric characteristic initial data problem for the Einstein-Maxwell-scalar field system with a positive cosmological constant in the interior of a black hole, assuming an exponential Price law along the event horizon. More precisely, we construct generic characteristic data which, on the outgoing initial null hypersurface (taken to be the event horizon), converges exponentially to a reference Reissner-Nördstrom black hole at infinity. We prove the stability of the radius function at the Cauchy horizon, and show that, depending on the decay rate of the initial data, mass inflation may or may not occur. In the latter case, we find that the solution can be extended across the Cauchy horizon with enough regularity to violate the Christodoulou-Chruściel version of strong cosmic censorship. This is joint work with J. Costa, J. Natário and J.D. Silva.

**Hugo Tavares** (CMAFCIO \ Faculdade de Ciências)

*Some aspects about reaction-diffusion systems with cooperative or competitive interactions*

In this talk, we will deal with systems of reaction-diffusion equations where the interaction between different components is either cooperative or competitive. Our aim will be to explain some of the relevant questions that can be asked for each type of interaction, explaining as well the motivations for its study. We will survey some of the results proved in the last decade, discussing in general the existence and characterization of positive solutions. Furthermore, we will explain how a strong competition induces a segregation phenomenon, and gives rise to a free boundary problem. In the last part of the talk, we will consider non-local interaction terms between the components. We will highlight some of the similarities and differences between the local and the non-local cases, showing some recent results in the non-local one, as well as some open problems.